

Understanding Pentaquark States in QCD

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We estimate the mass of the pentaquark state with QCD sum rules and find that pentaquark states with isospin $I = 0, 1, 2$ lie close to each other around (1.55 ± 0.15) GeV. The experimentally observed baryon resonance $\Theta^+(1540)$ with $S = +1$ can be consistently identified as a pentaquark state if its $J^P = \frac{1}{2}^-$. Such a state is expected in QCD. If its parity is positive, this pentaquark state is really exotic. Now the outstanding issue is to determine its quantum numbers experimentally.

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Quantum Chromodynamics (QCD) is believed to be the underlying theory of the strong interaction. In the high energy regime, QCD has been tested up to 1% level. In the low energy sector, QCD is highly nonperturbative due to the non-abelian $SU_c(3)$ color group structure. It is very difficult to calculate the whole hadron spectrum from first principles in QCD. With the rapid development of new ideas and computing power, lattice gauge theory may provide the final solution to the spectrum problem in the future. But now, people have just been able to understand the first orbital and radial excitations of the nucleon with lattice QCD in the baryon sector [1].

Under such a circumstance, various models which are QCD-based or incorporate some important properties of QCD were proposed to explain the hadron spectrum and other low-energy properties. Among them, it is fair to say that quark model has been the most successful one. It is widely used to classify hadrons and calculate their masses, static properties and low-energy reactions [2]. According to quark model, mesons are composed of a pair of quark and anti-quark while baryons are composed of three quarks. Both mesons and baryons are color singlets. Most of the experimentally observed hadrons can be easily accommodated in the quark model. Any state with the quark content other than $q\bar{q}$ or qqq is beyond quark model, which is termed as non-conventional or exotic. For example, it is hard for $f_0(980)/a_0(980)$ to find a suitable position in quark model. Instead it could be a kaon molecule or four quark state [3].

However, besides conventional mesons and baryons, QCD itself does not exclude the existence of the non-conventional states such as glueballs (gg, ggg, \dots), hybrid mesons ($q\bar{q}g$), and other multi-quark states ($qq\bar{q}\bar{q}, qqqq\bar{q}, qq\bar{q}\bar{q}\bar{q}, qqqqq, \dots$). In fact, hybrid mesons can mix freely with conventional mesons in the large N_c limit [4]. In the early days of QCD, Jaffe proposed the H particle [5] with MIT bag model, which was a six quark state. Unfortunately it was not found experimentally.

In the past years there have accumulated some experimental evidence of possible existence of glueballs and hybrid mesons with exotic quantum numbers like $J^{PC} = 1^{-+}$ [3]. Recently BES collaboration observed a possible signal of a proton anti-proton baryonium in the J/Ψ radiative decays [6]. But none of these states has been pinned down without controversy.

Recently LEPS Collaboration at the SPring-8 facility

in Japan observed a sharp resonance Θ^+ at 1.54 ± 0.01 GeV with a width smaller than 25 MeV and a statistical significance of 4.6σ in the reaction $\gamma n \rightarrow K^+ K^- n$ [7]. This resonance decays into $K^+ n$, hence carries strangeness $S = +1$. Later DIANA Collaboration at ITP observed the same resonance at 1539 ± 2 MeV with a width less than 9 MeV in a different reaction $K^+ Xe \rightarrow \Theta^+ Xe' \rightarrow K^0 p Xe'$ [8]. Now Θ^+ decays into $K^0 p$. The convincing level is 4.4σ . Very recently CLAS Collaboration in Hall B at JLAB also observed Θ^+ in the $K^+ n$ invariant mass at 1542 ± 5 MeV in the exclusive measurement of the $\gamma d \rightarrow K^+ K^- p n$ reaction [9]. The statistical significance is 5.3σ . The measured width is 21 MeV, consistent with CLAS detector resolution. The isospin, parity and angular moment of the Θ^+ particle has not been determined rigorously yet. There is only preliminary hint that Θ^+ might be an iso-singlet from the featureless $M(K^+ p)$ spectrum in the CLAS measurement.

Diakonov et al. proposed the possible existence of the $S = 1$ $J^P = \frac{1}{2}^+$ resonance at 1530 MeV with a width less than 15 MeV using the chiral soliton model [10], which partly motivated the recent experimental search of this particle. The authors argued that Θ^+ is the lightest member of the anti-decuplet multiplet which is the third rotational state of the chiral soliton model. Assuming that the $N(1710)$ is a member of the anti-decuplet, Θ^+ mass is fixed with the symmetry consideration of the model. Gao and Ma also discussed the experimental search of this pentaquark state [11]. Recently Polyakov and Rathke suggested possible photo-excitation of the pentaquark states [12].

Capstick, Page and Winston pointed out that identifying $N(1710)$ as a member of the anti-decuplet in the chiral soliton model is kind of arbitrary [13]. Instead, if the anti-decuplet P_{11} is $N(1440)$, the Θ^+ would be stable as the ground state octet with a very low mass while Θ^+ would be very broad with the anti-decuplet P_{11} being $N(2100)$ [13]. Furthermore, if the decay width of the anti-decuplet $N(1710)$ was shifted upwards to be comparable with PDG values, the predicted width of Θ^+ particle would have exceeded the present experimental upper bound [13]. The authors hypothesized that Θ^+ could be an isotensor resonance with negative parity [13]. The isospin violating decay width is naturally very small. Hence Θ^+ is a narrow state. Note similar isospin violat-

ing decay mechanism makes $D_{sJ}(2317)$ and $D_{sJ}(2460)$ states very narrow [14–16].

Stancu and Riska studied the stability of the strange pentaquark state assuming a flavor-spin hyperfine interaction between quarks in the constituent quark model [17]. They suggested that the lowest lying p-shell pentaquark state with positive parity could be stable against strong decays if the spin-spin interaction between the strange antiquark and up/down quark was strong enough [17]. The group classification of strange pentaquarks is given in [18].

In Ref. [19] Hosaka emphasized the important role of the hedgehog pion in the strong interaction dynamics which drives the formation of the Θ^+ particle and the detailed energy level ordering. Hyodo, Hosaka and Oset suggested measuring the Θ^+ quantum numbers using the reaction $K^+p \rightarrow \pi^+K^+n$ [20].

In Ref. [21] Karliner and Lipkin discussed the dynamics of the diquark-triquark pentaquark state with a rough estimate of the mass of 1592 MeV and $I = 0, J^P = \frac{1}{2}^+$ in the constituent quark model. The same authors discussed pentaquarks containing a heavy baryon [22].

Very recently Jaffe and Wilczek suggested that the observed Θ^+ state could be composed of an anti-strange quark and two highly correlated up and down quark pairs arising from strong color-spin correlation force [23]. The resulting J^P of Θ^+ is $\frac{1}{2}^+$.

Due to the low mass of Θ^+ , we think its angular momentum is likely to be one half. In this paper we shall employ QCD sum rules to estimate the mass of the pentaquark states with $J = \frac{1}{2}$ and $I = 0, 1, 2$.

The method of QCD sum rules incorporates two basic properties of QCD in the low energy domain: confinement and approximate chiral symmetry and its spontaneous breaking. One considers a correlation function of some specific interpolating currents with the proper quantum numbers and calculates the correlator perturbatively starting from high energy region. Then the resonance region is approached where non-perturbative corrections in terms of various condensates gradually become important. Using the operator product expansion, the spectral density of the correlator at the quark gluon level can be obtained in QCD. On the other hand, the spectral density can be expressed in term of physical observables like masses, decay constants, coupling constants etc at the hadron level. With the assumption of quark hadron duality these two spectral densities can be related to each other. In this way one can extract hadron masses etc. For the past decades QCD sum rules has proven to be a very powerful and successful non-perturbative method [24,25].

We use the following interpolating current for the $I = 2$ pentaquark state where three quarks and the remaining $q\bar{q}$ pair are both in a color adjoint representation. We note in passing such a choice of color configuration is not unique [26].

$$\eta_2(x) = \epsilon^{abc}[u_a^T(x)C\gamma_\mu u_b(x)]\gamma^\mu\gamma_5 u_e(x)\bar{s}_e(x)i\gamma_5 u_c(x) \quad (1)$$

for a charge $Q = +3$ state. Or

$$\eta_2(x) = \frac{1}{\sqrt{2}}\{\epsilon^{abc}[u_a^T(x)C\gamma_\mu u_b(x)]\gamma^\mu\gamma_5 d_e(x) \times \bar{s}_e(x)i\gamma_5 d_c(x) + (u \leftrightarrow d)\} \quad (2)$$

for a charge $Q = +1$ state. The isospin of the current is shown in its lower index. a, b, c etc are the color indices. T denote transpose. C is the charge conjugation matrix. The property $(C\gamma_\mu)^T = C\gamma_\mu$ ensures the current is symmetric under the exchange of the two quark fields inside the bracket. Both currents in Eq. (1) and (2) carry an isospin of two.

We want to emphasize the isospin and color structure of these currents guarantee they will never couple to a K^+n molecule or any other K^+n intermediate states since the isospin of a K^+n system can only be zero or one. One may worry about $K\Delta$ intermediate states since their total isospin can be two. However the energy of $K\Delta$ intermediate states are greater than $m_K + m_\Delta = 1726$, which is much larger than observed Θ^+ mass. In some cases, if the energy of the S-wave intermediate state is significantly lower than the resonance, its contribution could turn out to be quite important [27,16]. Angular momentum and parity conservation requires that only D-wave $K\Delta$ intermediate states could contribute if J^P of Θ^+ is $\frac{1}{2}^-$. When its J^P is $\frac{1}{2}^+$, then the $K\Delta$ combination should be of p-wave. Either P-wave or D-wave $K\Delta$ continuum contribution is negligible compared with the lower Θ^+ narrow resonance contribution. So these currents couple to $I = 2$ strange pentaquark states.

For $I = 0$ pentaquark state the interpolating current is

$$\eta_0(x) = \frac{1}{\sqrt{2}}\epsilon^{abc}[u_a^T(x)C\gamma_5 d_b(x)]\{u_e(x) \times \bar{s}_e(x)i\gamma_5 d_c(x) - (u \leftrightarrow d)\} \quad (3)$$

where $(C\gamma_5)^T = -C\gamma_5$ ensures the isospin of the up and down quark pair inside the first bracket to be zero. The anti-symmetrization in the second bracket ensures that the isospin of the other up and down quark pair is also zero.

For $I = 1$ pentaquark state the interpolating currents are

$$\eta_1(x) = \frac{1}{\sqrt{2}}\epsilon^{abc}[u_a^T(x)C\gamma_\mu d_b(x)]\{\gamma^\mu\gamma_5 u_e(x) \times \bar{s}_e(x)i\gamma_5 d_c(x) - (u \leftrightarrow d)\} \quad (4)$$

or

$$\eta_1(x) = \frac{1}{\sqrt{2}}\epsilon^{abc}[u_a^T(x)C\gamma_5 d_b(x)]\{u_e(x) \times \bar{s}_e(x)i\gamma_5 d_c(x) + (u \leftrightarrow d)\} \quad (5)$$

In the following we use currents (2), (3) and (5) to perform the calculation.

We introduce the overlapping amplitude f_j of the interpolating currents with the corresponding pentaquark states.

$$\langle 0 | \eta_j(0) | p, I = i \rangle = f_j u(p) \quad (6)$$

where $u(p)$ is the Dirac spinor of pentaquark field with $I = j$.

We consider the correlator

$$i \int d^4x e^{ipx} \langle 0 | T(\eta_j(x), \bar{\eta}_j(0)) | 0 \rangle = \Pi(p) \hat{p} + \Pi'(p) \quad (7)$$

where $\bar{\eta} = \eta^\dagger \gamma_0$ and $\hat{p} = p_\mu \cdot \gamma^\mu$. Throughout this note we focus on the chirality even structure $\Pi(p)$. At the hadron level it can be written as

$$\Pi(p) = \frac{f_j^2}{p^2 - M_j^2} + \text{higher states} \quad (8)$$

where M_j is the pentaquark mass.

On the other hand, it will be calculated in terms of quarks and gluons. For example, for the $I = 2$ case, the correlator reads

$$\begin{aligned} i \int d^4x e^{ipx} \{ & -2\epsilon^{abc}\epsilon^{a'b'c'} \text{Tr} [iS_{aa'}(x) \cdot \gamma_\nu C \cdot iS_{bb'}^T(x) \cdot C\gamma_\mu] \\ & \times \text{Tr} [i\gamma_5 \cdot iS_{e'e}^s(-x) \cdot i\gamma_5 \cdot iS_{cc'}(x)] \gamma^\mu \gamma_5 \cdot iS_{ee'}(x) \cdot \gamma^\nu \gamma_5 \\ & + 2\epsilon^{abc}\epsilon^{a'b'c'} \text{Tr} [iS_{aa'}(x) \cdot \gamma_\nu C \cdot iS_{bb'}^T(x) \cdot C\gamma_\mu] \\ & \times \gamma^\mu \gamma_5 \cdot iS_{e'e'}(x) \cdot i\gamma_5 \cdot iS_{e'e}^s(-x) \cdot i\gamma_5 \cdot iS_{cc'}(x) \cdot \gamma^\nu \gamma_5 \\ & + 4\epsilon^{abc}\epsilon^{a'b'c'} \gamma^\mu \gamma_5 \cdot iS_{ea'}(x) \cdot \gamma_\nu C [iS_{bb'}(x) \cdot i\gamma_5 \times \\ & iS_{e'e}^s(-x) \cdot i\gamma_5 \cdot iS_{cb'}(x)]^T C\gamma_\mu \cdot iS_{ae'}(x) \cdot \gamma^\nu \gamma_5 \} \end{aligned}$$

where $iS_{e'e}^s(-x)$ is the strange quark propagator in the coordinate space.

After making Fourier transformation to the above equation and invoking Borel transformation to Eq. (7) we get

$$f_j^2 e^{-\frac{M_j^2}{T^2}} = \int_{m_s^2}^{s_0} e^{-\frac{s}{T^2}} \rho_j(s) \quad (9)$$

where m_s is the strange quark mass, $\rho_j(s)$ is the spectral density and s_0 is the threshold parameter used to subtract the higher state contribution with the help of quark-hadron duality assumption. Roughly speaking $\sqrt{s_0}$ is around the first radial excitation mass. The spectral density reads

$$\begin{aligned} \rho_2 &= \frac{s^5}{4 \cdot 67! \pi^8} + \frac{s^3}{2 \cdot 136! \pi^8} \langle g_s^2 G^2 \rangle + \frac{s^2}{1536 \pi^4} \left[\frac{1}{3} \langle \bar{q}q \rangle^2 \right. \\ &\quad \left. + \frac{7}{6} \langle \bar{q}q \rangle \langle \bar{s}s \rangle \right] + \frac{5}{108} \langle \bar{q}q \rangle^3 \langle \bar{s}s \rangle \delta(s) \\ \rho_1 &= \frac{5s^5}{2 \cdot 177! \pi^8} + \frac{s^2}{1536 \pi^4} \left[\frac{13}{48} \langle \bar{q}q \rangle^2 + \frac{1}{3} \langle \bar{q}q \rangle \langle \bar{s}s \rangle \right] \end{aligned}$$

$$\begin{aligned} &+ \left[\frac{1}{108} \langle \bar{q}q \rangle^3 \langle \bar{s}s \rangle - \frac{1}{576} \langle \bar{q}q \rangle^4 \right] \delta(s) \\ \rho_0 &= \frac{3s^5}{4 \cdot 87! \pi^8} + \frac{s^2}{1536 \pi^4} \left[\frac{5}{12} \langle \bar{q}q \rangle^2 + \frac{11}{24} \langle \bar{q}q \rangle \langle \bar{s}s \rangle \right] \\ &+ \left[\frac{7}{432} \langle \bar{q}q \rangle^3 \langle \bar{s}s \rangle + \frac{1}{864} \langle \bar{q}q \rangle^4 \right] \delta(s) \end{aligned}$$

where we have used the factorization approximation for the multi-quark condensates.

In order to extract M_j we first take derivative of Eq. (9) with respect to $1/T^2$. Then we divide it by Eq. (9) to get

$$M_j^2 = \frac{\int_{m_s^2}^{s_0} ds e^{-s/T^2} \rho'(s)}{\int_{m_s^2}^{s_0} ds e^{-s/T^2} \rho(s)} \quad (10)$$

with $\rho'(s) = s\rho(s)$ except that $\rho'(s)$ does not contain the last term in $\rho(s)$.

In the numerical analysis of the sum rules the values of various QCD condensates are $\langle \bar{s}s \rangle = -(0.8 \pm 0.1) * (0.24 \text{ GeV})^3$, $\langle g_s^2 GG \rangle = 0.48 \text{ GeV}^4$. We use $m_s(1 \text{ GeV}) = 0.15 \text{ GeV}$ for the strange quark mass in the \overline{MS} scheme.

Numerically we have $M_2 = (1.53 \pm 0.15) \text{ GeV}$, $M_1 = (1.59 \pm 0.15) \text{ GeV}$, $M_0 = (1.56 \pm 0.15) \text{ GeV}$, where the central value corresponds to $T = 2 \text{ GeV}$ and $s_0 = 4 \text{ GeV}^2$. The variation of M_2, M_0 with both T and s_0 is shown in Figures (1)-(2), which contributes to the errors of the extracted value, together with the truncation of the operator product expansion, the uncertainty of vacuum condensate values and the factorization approximation of the multi-quark condensates. In the working interval of the Borel parameter M_j is reasonably stable with T . However the level ordering of the pentaquarks with isospin should not be taken too seriously due to the large uncertainty.

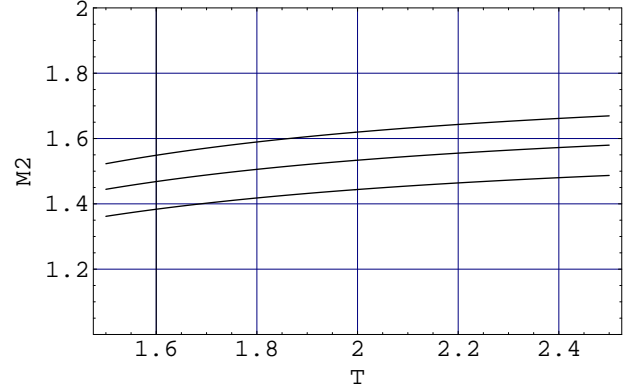


FIG. 1. The variation of the $I = 2$ pentaquark state mass M_2 with the Borel parameter T (in unit of GeV) and the continuum threshold s_0 . From bottom to top the curves correspond to $s_0 = 3.61, 4.0, 4.41 \text{ GeV}^2$ respectively.

Our results suggest that strange pentaquark states with $I = 0, 1, 2$ lie close to each other, which is very different from the prediction of chiral soliton model where

only the lightest $I = 0$ member is around 1540 MeV and all other states are several hundred MeV higher [10]. As we mentioned before, our interpolating currents can couple to pentaquark states with either positive or negative parity. QCD sum rules approach only picks out the lowest mass pentaquark state in the specific channel. It is impossible to determine its parity in our formalism.

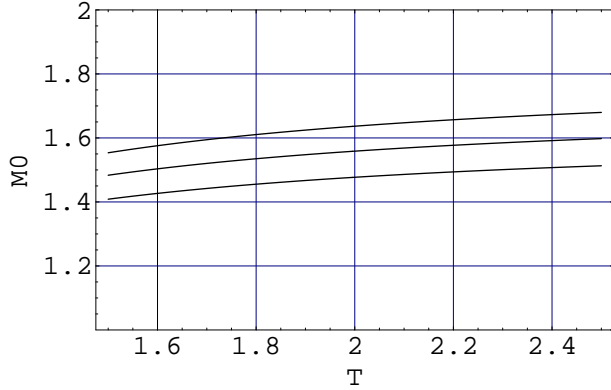


FIG. 2. The variation of the $I = 0$ pentaquark state mass M_0 with the Borel parameter T (in unit of GeV) and the continuum threshold s_0 . From bottom to top the curves correspond to $s_0 = 3.61, 4.0, 4.41$ GeV² respectively.

One may tend to think that Θ is a $J^P = \frac{1}{2}^-$ strange pentaquark state from previous quark model experience. One radial or orbital excitation of constituent quarks typically carries energy around 400 – 500 MeV. Naively one would expect the strange pentaquark mass with positive parity to be around $4M_u + M_s + 400 \sim 2050$ MeV, where $M_u = 300$ MeV and $M_s = 450$ MeV are roughly the up and strange quark constituent mass. The additional ~ 400 MeV comes from one orbital excitation for the positive-parity Θ^+ . If future experiments further establish the Θ^+ parity to be positive, then this particle is really very exotic. Otherwise, a $J^P = \frac{1}{2}^-$ Θ^+ particle is an expected strange pentaquark state in QCD. The outstanding issue is to measure the quantum numbers of the Θ^+ particle, especially the parity, angular momentum and isospin in the future experiments.

As a byproduct, we want to mention that the radial excitation Θ' of the strange pentaquark may lie around 2.0 GeV. If Θ is an iso-tensor state as suggested in [13], Θ' should mainly decay into final states containing one or two pions like $\Theta' \rightarrow K^+\Delta \rightarrow \pi^0 K^+ n$, $\Theta' \rightarrow \Theta\pi\pi \rightarrow K^+ n\pi\pi$, which conserve isospin symmetry. The isospin conserving decay mode $\Theta \rightarrow \pi^0 K^+ n$ is kinematically forbidden for an isotensor pentaquark. If its isospin is 0 or 1, then the dominant mode should be $\Theta' \rightarrow K^+ n$, $\Theta' \rightarrow \Theta\pi\pi \rightarrow K^+ n\pi\pi$. In other words, the decay mode $\Theta' \rightarrow \pi^0 K^+ n$ with a single pion in the final state can be used to distinguish the isospin of the strange pentaquark if enough events are collected in the future experiments.

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